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# Some Remarks on Supergiant Photospheres

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# Some remarks on supergiant photospheres

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Some characteristic aspects of near ultraviolet spectra of supergiant stars, observed with the Utrecht ultraviolet stellar spectrometer S59 aboard E.S.R.O.'s TD1A satellite are described. A comparison of the observed maximum brightness of supergiants with theoretical computation shows that in the brightest early-type supergiants the outward driving forces due to radiation pressure and to the turbulent pressure gradient must be about equal.

## 1. Observations of supergiants with S59

The greatest part of the observations of supergiant spectra accumulated so far with the orbiting spectrometer S59 aboard E.S.R.O.s TD1A satellite are in the process of discussion. Actually, part of the observations is still to be received from E.S.O.C. (April 1974) and most of the average spectra have to be deduced from the material yet streaming in. Therefore this paper has partly the character of an *interim report*. However, a few results obtained so far follow:

Figure 1*a–c* show some characteristic differences between spectra of main sequence stars and supergiants (Lamers *et al.* 1974), as observed in the three wavelength bands of the satellite.

Figure 2, from the same publication, refers to the equivalent widths of Fe III and Fe II lines. The supergiants show an abnormal behaviour as compared to more compact objects: the effect being mainly due to the large $W_\lambda$ values of the Fe III lines, which reflects the shift in the ionization balance due to the low density in supergiant atmospheres.

## 2. The effective acceleration of gravity in extreme supergiant and emission line stars

Figure 3, mainly after Th. and J. Walraven (1971) gives a plot of $E_{\text{B-U}}$ values against $E_{\text{V-B}}$ values for stars in the Magellanic Clouds, as compared with model calculations for stellar photospheres, by Mihalas and by De Jager & Neven. The thick lines indicate effective lg $g$ values of 2 and 1, respectively and the straight thin line is the $E_{\text{B-U}}$–$E_{\text{V-B}}$ relation for a black-body, of which the authors assumed that it would correspond with the energy–wavelength curve of a star with zero acceleration of gravity. The Walravens have deduced two conclusions from this diagram. First, the plots of the actual colours of supergiants in the Magellanic Clouds (not shown in this figure) indicate that effective $g$ values of such stars are very small, with lg $g_{\text{eff}} \approx 1$ as a characteristic value.

The star HD 33579 (A3, Ia-O(e)), the most luminous non-variable star known, is an example. From an extrapolation of the lg $g$ = constant curves the Walravens estimate that for this object lg $g_{\text{eff}} \approx 0$, which is confirmed by the observationally determined value lg $P_{\text{e}} = 0$ (Przybylski 1968). In agreement with this, Wolf (1972) found from a detailed spectral investigation of this star that lg $P_{\text{e}} = 0$ at $\bar{\tau} \approx 0.35$. He determined a Newtonian acceleration of

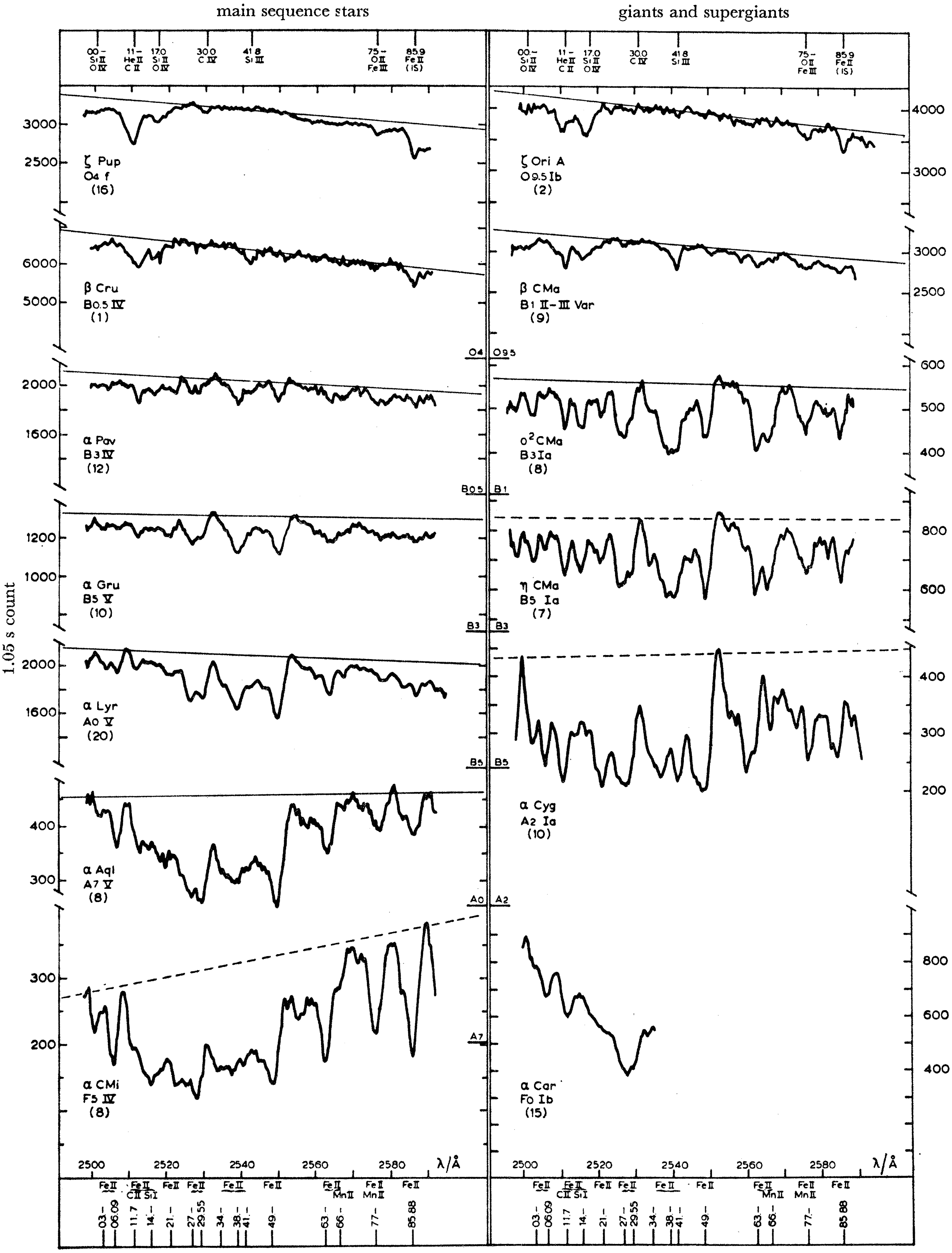


FIGURE 1. Characteristic S59 spectra, showing the differences between main sequence stars and supergiants (Lamers *et al.* 1974).

gravity $\lg g_N = 0.7$; no $g_{eff}$ value was communicated. The positions of early-type emision line stars are indicated in the diagram by triangles. Since they occur virtually *on* the black-body line this suggests that $g_{eff} \approx 0$ for such kinds of stars. This means actually

with
$$\left.\begin{aligned} g_{rad} + g_{turb} + g_N &\approx 0, \\ g_N = G\mathfrak{M}/R^2; \quad g_{rad} &= \frac{1}{\rho}\frac{dP_{rad}}{dz} = -\kappa\pi F/c; \\ g_{turb} &= \frac{1}{\rho}\frac{dP_{turb}}{dz}. \end{aligned}\right\} \quad (1)$$

Here $\kappa$ is the extinction coefficient, $z$ the vertical ordinate, the other symbols have their usual meaning.

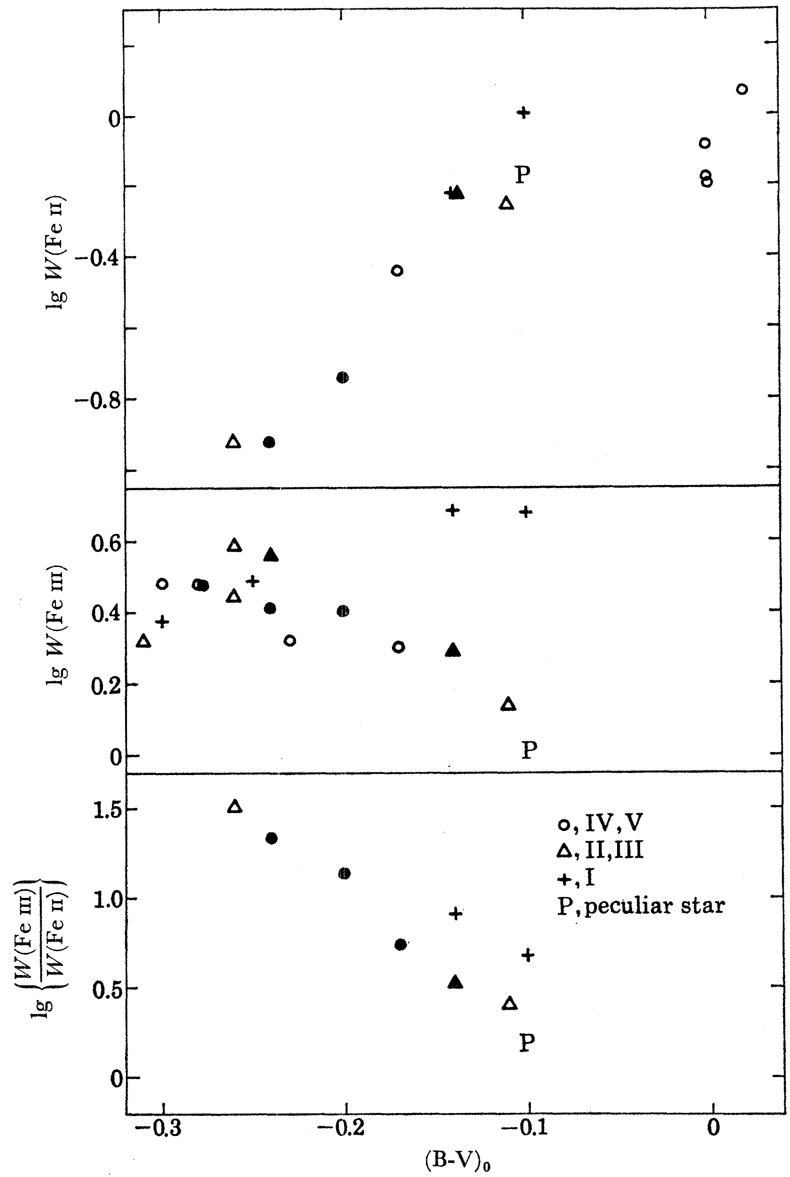


FIGURE 2. Equivalent widths of near ultraviolet Fe II and Fe III lines in stars (Lamers *et al.* 1974).

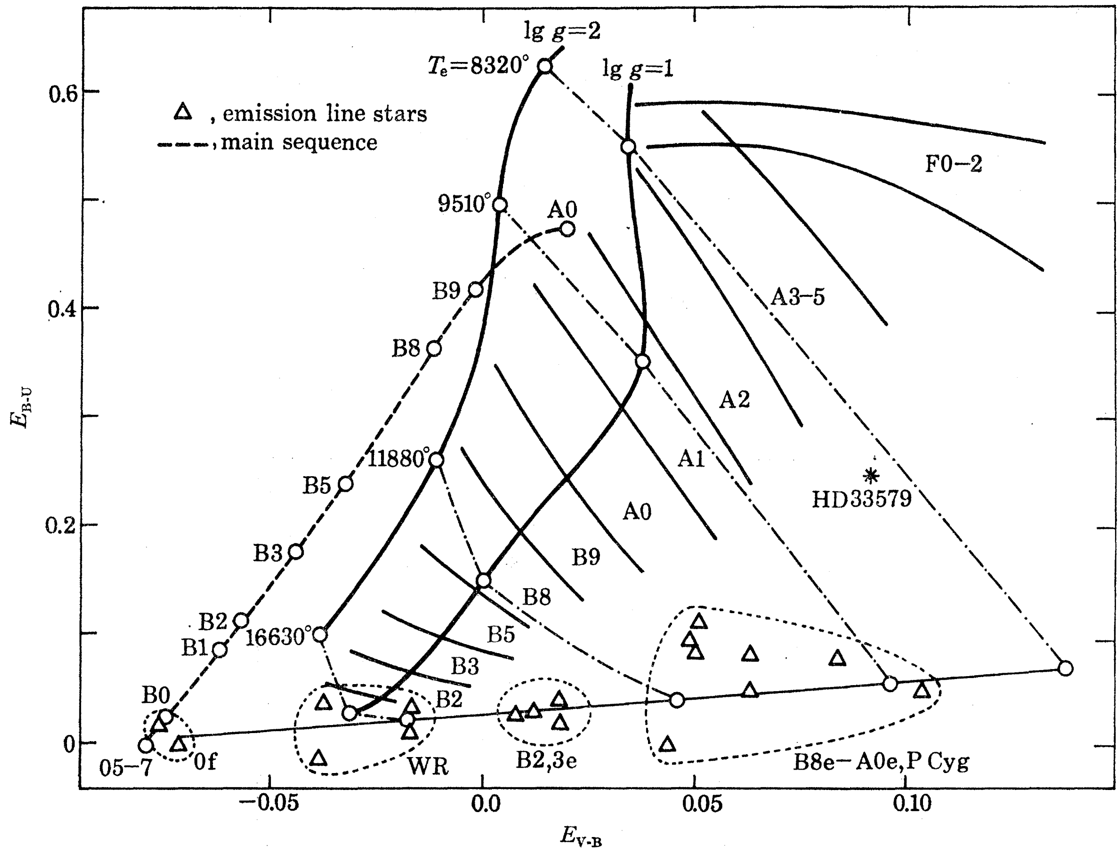


FIGURE 3. $E_{B-U}$–$E_{V-B}$ diagram for stars in the Magellanic Clouds, after Th. & J. Walraven (1971). The extreme supergiant HD 33 579 has a very small $g_{eff}$ value, estimated at *ca* 1 cm² s⁻¹. Emission line objects seem to have $g_{eff} \approx 0$.

## 3. THE MAXIMUM LUMINOSITY OF SUPERGIANTS

The Hertzsprung–Russell diagram for bright stars (figure 4) shows a maximum bolometric absolute magnitude $M_{lim} \approx -9$ to $-10$. Already Eddington (1921) estimated the maximum possible luminosity of stars, by equalizing the gravitational acceleration and that due to radiation pressure. We repeat his calculations. By taking equation (1) and writing $\pi F = L/4\pi R^2$; $l = L/L_\odot$; $m = \mathfrak{M}/\mathfrak{M}_\odot$, $\mathfrak{M}_\odot = 1.99 \times 10^{33}$ g; $L_\odot = 3.90 \times 10^{26}$ J s$^{-1}$ we obtain:

$$l = 1.28 \times 10^4\, m/\kappa. \tag{2}$$

With a stellar mass-luminosity relation $$l = m^n, \tag{3}$$

equation (2) reduces to $$l = (1.28 \times 10^4/\kappa)^{n/(n-1)}. \tag{4}$$

For the upper end of the main sequence the value of $n$ is not very certain but 3.7 seems reasonable. One then gets (De Jager 1975):

| | | | | | |
|---|---|---|---|---|---|
| for $T_e$ = | 3700 | 5660 | 9733 | 23 425 | 32 300 K |
| $(M_{bol})_{lim}$ = | −18 | −12.5 | −11.4 | −11.2 | −11.2 magnitudes |

For the cool stars the derived limiting magnitude deviates greatly from observations, but also for the hot stars the values calculate here exceed the observed limit by about one magnitude. The differences for the cool stars are partly due to the fact that equation (2) as written here does not apply to such stars: E. Böhm-Vitense (1973) has shown that in supergiants with $T_e <$ 8000 K non-grey effects, including line absorption raise $g_{rad}$ by a factor of at least 6. A computation shows that this effect would increase the value of the limiting magnitude by $+2.7$ magnitudes. This would not explain the difference for the *coolest* stars, for which a non-grey amplification factor of the radiation pressure of about 200 would be demanded. The actual reason why no cool supergiants exist with bolometric magnitudes $\lesssim -10$, is that because of the horizontal position of the evolutionary tracks the observed limiting brightness of cool supergiants *depends* on and is nearly equal to the value for hot stars. Hence no cool supergiants can exist with luminosities well exceeding those of the main sequence stars, and consequently the evolved reddest supergiants are still at a safe distance from their instability point, at variance with the hot objects. For the latter stars the difference between observations $M \approx -10$ and expectations $M \approx -11$ should be due either to a gradient in the turbulent pressure, or to increased radiation pressure due to lines or other non-grey effects.

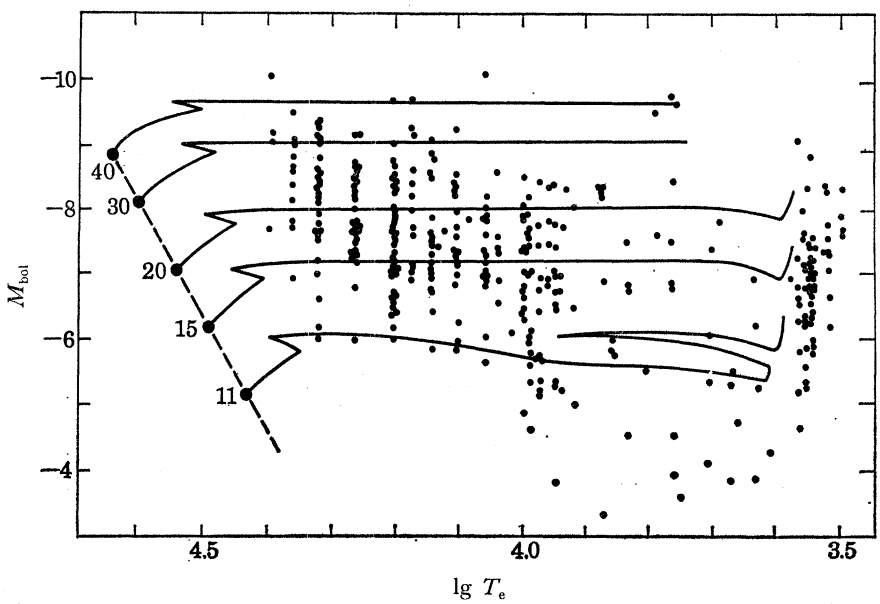


FIGURE 4. $M_{bol}$–lg $T_e$ diagram for supergiants, composed by Humphreys (1970). The broken line gives the theoretical main sequence, and the drawn lines are evolutionary tracks for stars of various masses.

With regard to the former effect (*turbulent pressure*) it is of importance to note that in the few stars for which reasonably accurate values are now available the turbulent pressure seems indeed to decrease outward, leading to an acceleration term that (partly) compensates the Newtonian gravitation. For instance, for the model of the A3Ia-O supergiant HD 33579 published by Wolf (1972) one finds that between $\bar{\tau} = 1$ and $0.1$: $g_{turb} \approx -0.2$, but between $\bar{\tau} = 0.1$ and $0.001$: $g_{turb} \approx -2.5$, nearly half the Newtonian value $g_N = 5$. This effect could be explained if the observed turbulence would be *generated* in deep layers, thereupon *propogates* outward and being largely *dissipated* in chromospheric regions, thus leading to an outward decrease of the turbulent pressure. If there were no dissipation, $g_{turb}$ would be zero. We therefore repeat the computations leading to equation (4) by writing

$$g_N + g_{turb} = f_1 g_N,$$

and

$$g_{rad} = -f_2 \kappa \pi F / c.$$



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It is then easily found that in order to have full compensation, by accounting for the difference $\Delta m$ between predicted and observed absolute bolometric magnitudes:

$$\lg\,(f_2/f_1) = -0.3\,\Delta m.$$

With $\Delta m = -1$, and $f_1 \approx 0.5$ (tentatively, after the data on HD 33579), one would find $f_2 = 1$, which would indicate that the radiation pressure due to lines should be practically negligible as compared to the effect of the continuous spectrum. Furthermore $g_{\text{rad}} \approx g_{\text{turb}}$.

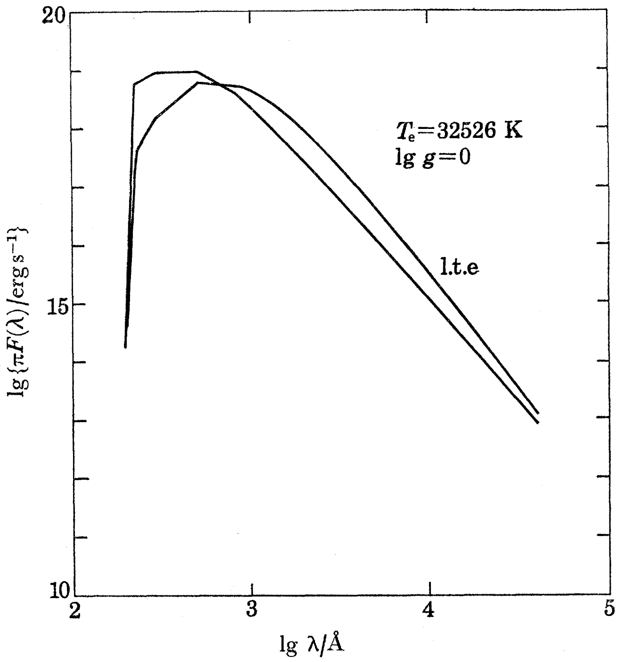


FIGURE 5. Emergent flux $\lg \pi F(\lambda)$ of a hot extreme supergiant ($T_e = 33\,000$ K, $\lg g = 0$), as compared to the flux obtained from

$$\int_0^\infty B(\lambda\tau_\lambda)\,\kappa_2(\tau_\lambda)\,\mathrm{d}\tau_\lambda,$$

showing the shortward displacement of the maximum of the emission curve for a scattering photosphere.

## 4. Source functions, fluxes and temperature stratification in extreme supergiants

Source function $S(\lambda, \tau_\lambda)$ and consequent emergent fluxes $\pi F(\lambda)$ were computed by De Jager & Neven (1975) for extreme supergiants ($\lg g = 0$, 0.5 and 1.0) with effective temperatures ranging between 3700 and 33000 K. Especially for the atmospheres of hot supergiants scattering at free electrons is the main extinction mechanism. This has two effects. First, since the scattering process is wavelength independent there is little or no redistribution of radiation over the wavelengths during the process of outward radiation transfer; this makes the shape of the $\pi F(\lambda)$ curve somewhat similar to the black-body curve for the deeper layers with higher temperatures.

Figure 5 shows that the maximum of the emission curve is shifted to short wavelengths, so that very hot supergiants may even appear to be observable by sensitive extreme u.v. detectors in satellites. Another aspect refers to the depth-dependence of the radiation flux. As a consequence of the large $\sigma/\kappa$ ratio the radiation flux $\pi F(\tau, \lambda)$ is nearly fully independent of the assumed temperature–depth relation. Actually, as trial calculations show one obtains for a given effective temperature nearly the *same* $S(\lambda, \tau_\lambda)$ curves for virtually *any* assumed $T(\tau, \lambda)$-relation, provided that $\kappa_\lambda/\sigma_\lambda \approx 0$. As a consequence, however, in such extreme cases, the temperature–depth relation in an extreme supergiant atmosphere can ultimately only stabilize to a situation in which the local kinetic temperature of the gas equals the local radiation temperature. It appears, however, that there is no unique radiation temperature at a given geometrical level, so that the kinetic temperature would stabilize to the weighted average of the radiation temperature (weighting factor: local source function). Semi-quantitatively this can be shown as follows (a more detailed discussion will appear elsewhere).

The physical process of energy exchange at collisions between photons and free electrons in the Compton process. The average energy exchange at one collision is

$$\Delta E \approx E\lambda/\lambda_c,$$

with

$$\lambda_c = h/mc = 2.4 \times 10^{-3}\ \text{nm}.$$

Hence, with $\bar{\lambda} = 30$ nm,

$$\Delta E/E \approx 10^4.$$

If this value should be reduced to half its value in order to equalize $T_{kin}$ with $T_{rad}$, then about 5000 collisions would be needed per electron. With a scattering cross section $6.6 \times 10^{-25}\ \text{cm}^2$, and a radiation density $\sigma T_e^4/c = 1.5 \times 10^{-4}\ \text{J cm}^{-3}$ (assuming $T_e = 30000$ K), one electron undergoes about two collisions per second, so that about $2 \times 10^3$ s are needed for the electron gas to adjust to the temperature of the radiation field.

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